A NEW METHOD FOR POWER SYSTEM QUALITY ANALYSIS BASED ON THE INSTANTANEOUS COMPLEX POWER THEORY

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Abstract-This paper enhances some concepts of the Instantaneous Complex Power Theory [1,2] by analyzing the analytical expressions for voltages, currents and powers developed on a symmetrical RL three-phase system, during the transient caused by a sinusoidal voltage excitation.

The powers delivered to an ideal inductor will be interpreted, allowing a deep insight in the power phenomenon by analyzing the voltages in each element of the circuit. The results can be applied to the understanding of non-linear systems subject to sinusoidal voltage excitation and distorted currents.

Keywords: Power quality, instantaneous complex power, electrical power systems.

I- INTRODUCTION

This paper shows the development and the analysis of analytical expressions for voltages, currents and power on a symmetrical RL three-phase system, during the transient caused by a sinusoidal voltage excitation. The basic concepts and definitions used here were presented on the so called Instantaneous Complex Power Theory [1,2]. This theory is based mainly in the fact that ideal inductors receive active and reactive powers and the imaginary powers is the envelope of the instantaneous reactive power.

Since the circuit voltage excitations are sinusoidal, the sum of the non-sinusoidal voltages on the resistive and inductive circuit elements is sinusoidal, accordingly to the Kirchoff voltage law, because there is a transient voltage drop component on each element that cancels one another. It can be verified that the real power, which is received by the ideal inductors, is related to these non-sinusoidal voltage drops which are produced by the transient current components on the resistive circuit elements.

It will be shown that the powers produced by the sinusoidal voltage component and the total current are those supplied by the ideal voltage source and the powers produced by the transient voltage and total current are those exchanged between the circuit elements.

Furthermore, it is suggested that the results of this analysis may be generalized for a better understanding of power systems in steady-state with non-sinusoidal waveforms, usually analyzed by Fourier methods. By this way, power systems operating on non-sinusoidal steady-state conditions must be considered as a sequence of transient periods.

As a matter of revision, it will be presented on the next section some concepts and power definitions found in the above mentioned Instantaneous Complex Power Theory.

II. THE INSTANTANEOUS COMPLEX POWER

The following transformation is general, but it will be considered in this paper only for the symmetrical case.

Consider v_a , v_b and v_c , which are the excitation voltages of a symmetrical three-phase RL circuit, given by:

$$v_a = \hat{V} sin\left(wt + \theta \right) \quad (1)$$

$$v_b = \hat{V} sin\left(wt + \theta - 120^o \right) \quad (2)$$

$$v_c = \hat{V} sin\left(wt + \theta + 120^o \right) \quad (3)$$

A instantaneous space vector is obtained from the following vectorial expression on the complex plane:

$$\tilde{V} = \frac{2}{3}\left(v_a + a\, v_b + a^2\, v_c \right) \quad (4)$$

where:

$$a = \exp\left(j\frac{2\pi}{3} \right) \quad (5)$$

The instantaneous space vector, $\tilde{I}$, for the current, is obtained in a similar way.

These vectors are shown in Fig. 1.

The exponential (or polar form) representations for these vectors are:

$$\tilde{V} = V\exp\left(j\phi_v\right) \tag{6}$$

$$\tilde{I} = I\exp\left(j\phi_I\right) \tag{7}$$

The expression (4) indicates an operation that transforms three-phase system into an equivalent two-phase system.

Fig. 2 shows the two-phase equivalent circuit where phase α is coincident to the phase a of the original three-phase system, and phase β is orthogonal to phase α.

The instantaneous complex power is defined for three-phase systems as [1]:

$$\tilde{S} = \frac{3}{2}\tilde{V}\,\tilde{I}^{*} \tag{8}$$

The real power is given by the real part of the instantaneous complex power as indicated in the following expression:

$$P = Real\left\{\tilde{S}\right\} \tag{9}$$

The imaginary power is given as the imaginary part:

$$Q = Imag\left(\tilde{S}\right) \tag{10}$$

The real power is the power exchanged between energy storage elements and external sources, which is related to the stored energy variation or to the power that flows through the system. The real power is obtained from the component of $\tilde{I}$ in phase with $\tilde{V}$ (see Fig. 3).

The imaginary power is a quantity proportional to the stored energy into the electrical circuit when the angular speed of the instantaneous current space vector is constant. When the stored energy is constant, the system is in steady-state and the imaginary power becomes the conventional reactive power [1]. It is also the envelop of the instantaneous reactive power (see Fig. 5). The imaginary power is obtained from the component of $\tilde{I}$ in quadrature with $\tilde{V}$.

III. ANALYTICAL EXPRESSIONS FOR VOLTAGES, CURRENTS AND POWERS.

The expression for voltages in a symmetrical three-phase system can be developed as follows:

$$\tilde{V} = R\tilde{I} + L\frac{d\tilde{I}}{dt} \tag{11}$$

As the excitation voltages are sinusoidal they can be represented by the following rotating vector of constant amplitude and speed:

$$\tilde{V} = \dot{V}\exp\left(j\omega t\right) \tag{12}$$

where

$$\dot{V} = \hat{V}\exp\left(j\alpha\right) \tag{13}$$

is the voltage phasor.

The current vector derivative is expressed as:

$$\frac{d\tilde{I}}{dt} = j\omega_I\,\tilde{I} + \frac{dI}{dt}\exp\left(j\phi_I\right) \tag{14}$$

where

$$\omega_I = \frac{d\phi_I}{dt} \tag{15}$$

is the instantaneous current vector angular speed. Substituting (14) into (11) it becomes:

$$\tilde{V} = R\tilde{I} + j\omega_I L\tilde{I} + L\frac{dI}{dt}\exp(\tag{16}$$

Using (11) to (16), the instantaneous complex power equation (8) can be written as:

$$\tilde{S} = \frac{3}{2}\left(RI^2 + j\omega_I LI^2 + LI\frac{dI}{dt}\right) \tag{17}$$

The first term of this expression is the real power dissipated on the resistors, the second and the last terms are the imaginary power and the real power, respectively, delivered to the ideal inductors. The expression for the stored energy is given as:.

$$\ell_L = \frac{3}{2}\int_0^I L\,I\;dI \tag{18}$$

or

$$\ell_L = \frac{3}{2}\,L\,I^2 \tag{19}$$

It can be noticed that the stored energy is related to the current vector magnitude.

Fig. 4 shows the real power, the imaginary power and the energy stored in the inductors during the transient period.

Fig. 5 shows the imaginary power as the envelop of phase β instantaneous reactive power; and Fig. 6 shows the phase β instantaneous power as the sum of the instantaneous active and reactive powers.

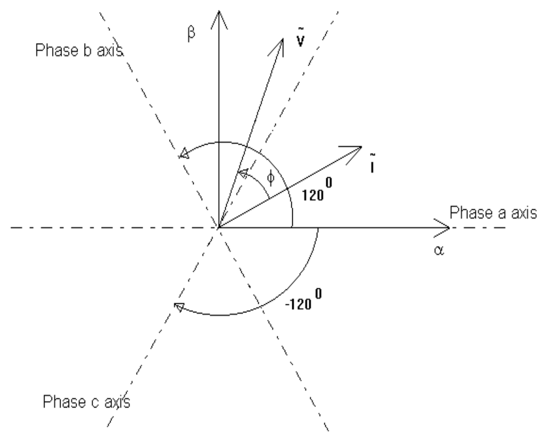

Fig.1-Voltage and current instantaneous space vectors representation on a complex plane.

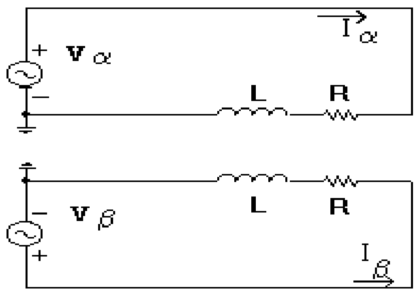

Fig.2-Equivalent two-phase representation of a symmetrical three-phase circuit (V=250 V, R=10 ohms, L=100mH).

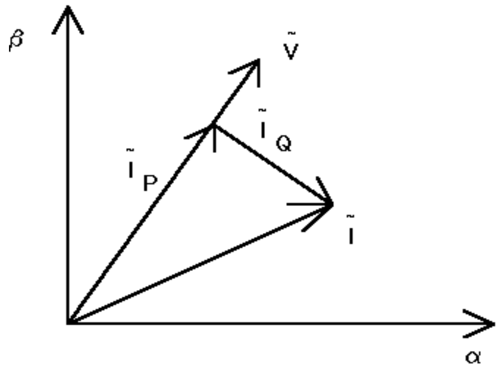

Fig. 3- Space vectors representation of the current projections in phase and in quadrature with voltage.

The expression for the transient current is obtained by the integration of the voltage in equation (11). This vectorial current expression is the representation of the current in each phase, by projecting it on axis of phases a, b and c, respectively (see Fig. 1):

$$\tilde{I} = \hat{I}\exp\left(j(\omega t+\alpha)\right) - \hat{I}\exp\left(\frac{-R}{L}t\right)\exp(j\alpha) \quad (20)$$

Then, substituting (20) into (16), the analytical expression for voltage on the load is given by:

$$\tilde{V} = R\hat{I}\exp\left[j(\omega t+\alpha)\right] - R\hat{I}\exp\left(\frac{-R}{L}t\right)\exp(j\alpha) + j\omega L\hat{I}\exp j(\omega t+\alpha) - L\hat{I}\exp(j\alpha)\frac{-R}{L}\exp\left(\frac{-R}{L}t\right) \quad (21)$$

The second and the last terms of (21) cancel each other, but they must be taken in account for obtaining the powers on the inductor and on the resistor.

Multiplying equation (21) by (20), the following expressions are obtained for the circuit elements:

Real power dissipated on the resistors:

$$P_R = R\hat{I}^2 - 2R\hat{I}^2\exp\left(\frac{-R}{L}t\right)\cos(\omega t) + R\hat{I}^2\exp\left(\frac{-2R}{L}t\right) \quad (22)$$

Imaginary power on the resistors:

$$Q_R = 0 \quad (23)$$

Real power delivered to the inductors:

$$P_L = \omega L\hat{I}^2\exp\left(\frac{-R}{L}t\right)\sin(\omega t) + R\hat{I}^2\exp\left(\frac{-R}{L}t\right)\cos(\omega t) - R\hat{I}^2\exp\left(\frac{-2R}{L}t\right) \quad (24)$$

Imaginary power delivered to the inductors:

$$Q_L = \omega L\hat{I}^2 - \omega L\hat{I}^2\exp\left(\frac{-R}{L}t\right)\cos(\omega t) - R\hat{I}^2\exp\left(\frac{-R}{L}t\right)\sin(\omega t) \quad (25)$$

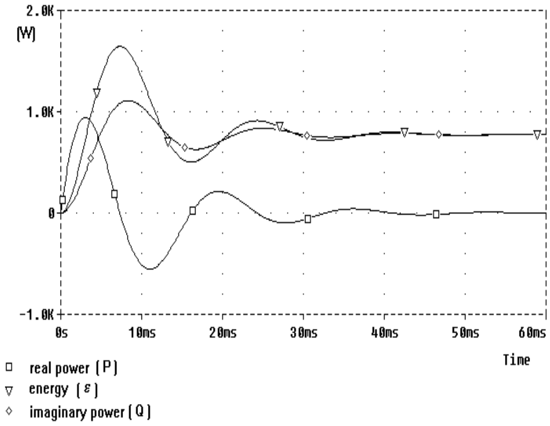

Fig. 4- Real and imaginary powers and energy received by the ideal inductors.

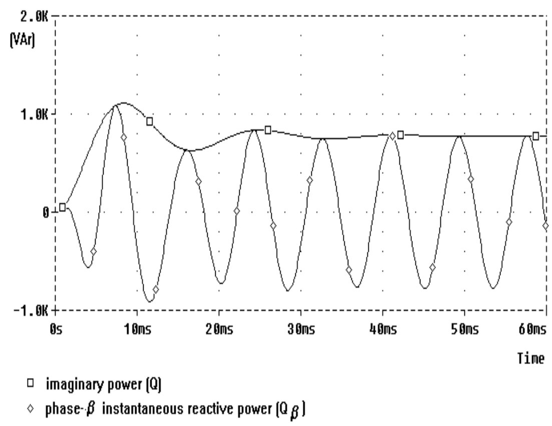

Fig. 5-Imaginary and instantaneous reactive powers for the phase- β inductor

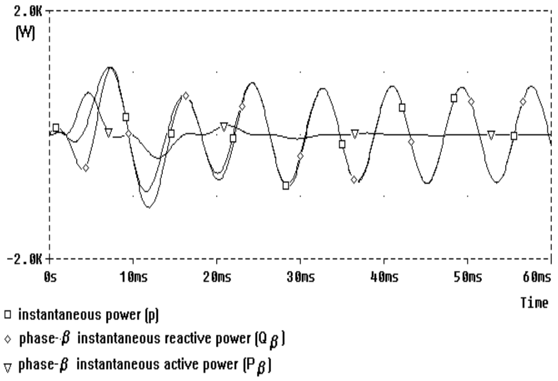

Fig. 6-Instantaneous power and instantaneous active and reactive powers, received by the phase- b ideal inductor**.**

The power supplied by the source is given by the expression:

$$P_S = R\hat{I}^2 - R\hat{I}^2 \exp\left(\frac{-R}{L}t\right)\cos(\omega t) + \omega L\hat{I}^2 \exp\left(\frac{-R}{L}t\right)\sin(\omega t) \quad (26)$$

It must be observed that the source doesn't "see" the power distribution on the load. This distribution is due to the transient voltages on each load component. In this way, it is impossible, from the source viewpoint, to know the actual value of the real power on the inductor.

A very important conclusion is that the steady-state voltage component (third term of (21)) across the inductor is responsible for the component of power delivered by the source, and the transient voltage component (the fourth term of (21)) is responsible for the power exchanged with the resistance.

It can be observed that as the time constant of the circuit depends on the resistance value, ideal inductors may take a long time to be loaded.

IV- STEADY-STATE OPERATION WITH SINUSOIDAL VOLTAGES AND NON-SINUSOIDAL CURRENTS

Consider the following expressions for the currents on the system without neutral conductor:

$$i_a = \sqrt{2} I_1 \,\text{sen}(\omega t - \Phi_1) + \sqrt{2} I_2 \,\text{sen}(2\omega t - \Phi_2) + \ldots \quad (27)$$

$$\text{i}_b = \sqrt{2}\,\text{I}_1 \,\text{sen}\left(\omega t + 120^0 - \Phi_1\right) + \sqrt{2}\,\text{I}_2 \,\text{sen}\left[2\left(\omega t + 120^0\right) - \Phi_2\right] + \cdots \quad (28)$$

$$\text{i}_c = \sqrt{2}\,\text{I}_1 \,\text{sen}\left(\omega t - 120^0 - \Phi_1\right) + \sqrt{2}\,\text{I}_2 \,\text{sen}\left[2\left(\omega t - 120^0\right) - \Phi_2\right] + \cdots \quad (29)$$

After the transformation indicated in (4), the components in the complex plane are given by:

$$I_a = \sum_{n=1}^{\infty} \frac{1}{\sqrt{3}} I_n \,\text{sen}\left(n\omega t - \Phi_n\right)\left[1 - \cos\left(n120^0\right)\right] \quad (30)$$

$$I_b = \sum_{n=1}^{\infty} 2\, I_n \cos\left(n\omega t - \Phi_n\right) \text{sen}\,(n12 \quad (31)$$

These two expressions can be represented by the following vectorial expression:

$$\tilde{I} = \sum_{k=1}^{\infty} \dot{I} \exp\left(j k \omega t\right) \quad (32)$$

and the harmonics can be represented on the complex plane as:

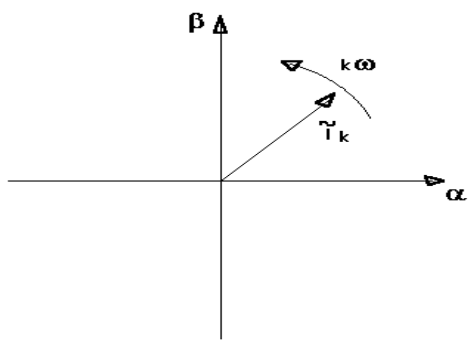

Fig. 7- Harmonics representation on the complex plan.

Where k is the harmonic order.

The instantaneous complex power can be expressed as the sum of an constant vector (with is the conventional complex power) and an rotating vector with speed and amplitude varying in the time. Given by the equation:

$$\tilde{S} = \bar{S} + \tilde{H} \quad (33)$$

Fig 8 shows their representation on the complex plan:

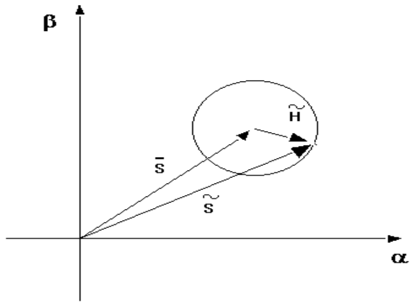

Fig. 8- Instantaneous complex power representation for non-linear systems.

These conclusions can lead to predict that, for power systems harmonic studies (in this case the system operation must be considered as a sequence of transients), when only the fundamental voltage is taken in account, the evaluated power delivered to the load is the power supplied by the source. It is necessary to notice that the voltage and current harmonics of different orders are taken in account with the instantaneous complex power analysis, and these harmonics produce real distortion powers and imaginary distortion powers in spite of their zero mean value (in conventional Fourier analysis, these instantaneous power aren't taken in account).

Recall that real distortion power is the power exchanged between electrical energy storage elements and machine shafts (when the source is an electrical machine), which

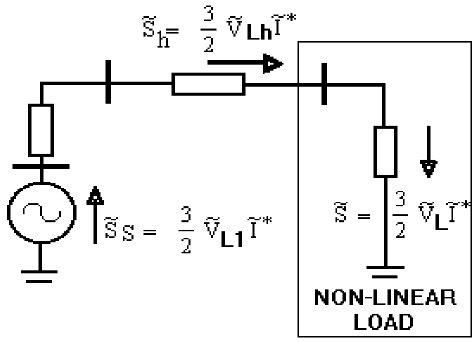

Fig. 9- Steady-state power flowing to a non-linear load.

produces vibration on mechanical equipment (mechanical power).

To evaluate the power transferred to the remaining system, it is necessary to consider the transient voltage on the load. It means that, by just measuring the power developed on the load, the power delivered by the source and the power transferred to the remaining system may be evaluated too.

Fig. 9 shows the power $\tilde{S}_S$ delivered by the source to the load and the power $\tilde{S}_h$ exchanged between the load and the remaining system (the source also supplies the remaining system):

V-CONCLUSIONS

Analytical expressions were developed for the powers delivered to a three-phase circuit (or to an equivalent two-phase), which allows a better understanding of the power in electrical systems.

The instantaneous power presented in this work may be thought as the actual energy power flow rate on the circuit element, rather than the approximation obtained using the Fourier approach.

For power quality studies, the Instantaneous Complex Power Theory promises to be a complement or an alternative to the Fourier theory. The idealized power system operates with sinusoidal waves with constant stored energy. These characteristics are present on balanced steady-state linear systems.

On power system analysis, it is possible to evaluate the real and the imaginary powers delivered by the source and the real and imaginary power transferred to the transmission system by a simple measurement on the load terminals. It may be very useful on power quality studies.

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REFERENCES

[1] D. L. MILANEZ & MISKULIN, M. S.,"The Instantaneous Complex Power Applied to Three-Phase Machines", Proc. 28 Industry Application Society-IEEE- Annual Meeting , pp. 171-176, 1993.

[2] MILANEZ,D.L. "New Concepts of the Power Received by Ideal Energy Storage Elements:The Instantaneous Complex Power Approach". IEEE-1996-International Symposium on Circuits and Systems- Ames,Yowa,USA, pp. 1038-1041.

[3] Akagi, H.; Kanazawa, Y. and Nabae, A.,"Instantaneous Reactive Power Compensator Comprising switching Devices Without Energy Storage components.", IEEE Transactions on Industry Applications, Vol. IA-20, n^o 3, May/June-pp.625-630.,1984.

[4] Milanez, D.L., Seixas, F.J.M.& Alves, D. A.,"A Composição da Potência Instantânea Recebida por Elementos Armazenadores de Energia Ideais", Proc. Seminá-rio Brasileiro De Qualidade Da Energia Elétrica-SBQEE-1996- Uberlândia-Minas Gerais-Brasil, pp. 186-189, 1996

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