A CRITICAL REVIEW OF THE TRANSFORMER MAGNETISING CHARACTERISTIC MODELS

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Abstract: Transformer magnetising currents can contribute substantially to the harmonic content of the electric power systems. Therefore, an accurate representation of the magnetising characteristic is important in harmonic studies. This paper examines the adequacy of the piecewise linear, hyperbola, rational-fraction and cubic spline approximations to represent the magnetising characteristic of the various magnetic branches of a five-limb core transformer. A procedure based on Singular Value Decomposition (SVD) is investigated as an alternative to determine the rational-fraction and hyperbola coefficients.

Keywords: Saturation characteristic, multilimb transformers, harmonics

1.INTRODUCTION

Modern transformer cores are designed to operate very close to the knee point of the saturation characteristic. As a consequence, even under small over excitation, the magnetising current contributes substantially to the harmonic content. Thus, an accurate representation of the saturation characteristic is of fundamental importance in harmonic studies.

For computational analysis, it is convenient to approximate them by an analytical expression, ideally, by a single and mathematically simple function able to reproduce the magnetising characteristics over their entire range with a minimum error. Due to the saturation and hysteresis properties exhibited by these curves the derivation of a mathematical function that is both accurate and simple is a difficult task. The problem can be simplified by neglecting hysteresis, a reasonable assumption in steady-state analysis and modern cold-rolled steel transformers [1].

There is a wide range of literature on approximations for magnetising characteristics. However, these approximations have limitations regarding either the range of magnetising characteristics that they can accurately reproduce or the determination of their equation coefficients.

Very close reproduction of a large range of magnetising characteristics can be obtained by the rational-fraction function [2]. However, a very complicated iterative procedure is required to calculate the equation coefficients which has inhibited the use of this formulation. The same level of accuracy can be obtained by a much simpler approach, the hyperbola function [1] [3]. A trial-and-error approach is required to calculate the variable associated with the smoothness of the knee region.

More recently, it has been stated in some publications, that two or three piecewise linear segments [1] [4][5](Discussions by R. A. Walling) are sufficient to represent the transformer magnetising characteristics for transient studies. However, an accurate representation of the entire curve might be needed for studies where resonance is an important factor [6]. Moreover, it has been reported in the literature that the 'kinks' caused by the junction of two straight lines have been magnified in the solution of a problem [2]. So far, this aspect has not been thoroughly investigated for steady-state harmonic analysis.

While most of the transformer electromagnetic models are based on the flux-current ϕi characteristic, the BH curve representation is more commonly described in the literature. Although the curve ϕi can be obtained by simply rescaling the BH curve by the core dimensions ($B=\phi.A$and $H=N.i.l$), the sharpness of the knee region of a ϕi characteristic will depend on the design; this can compromise the adequacy of some mathematical equations describing BH curves to represent ϕi characteristics.

The purpose of this paper is to analyse the importance of an accurate representation of the magnetising characteristic in harmonic steady-state studies for resonant systems. It also analyses the adequacy of piecewise linear, hyperbola and rational-fraction approximations for different magnetising characteristics of a five-limb core transformer. A procedure based on Singular Value Decomposition (SVD) is investigated as an alternative to determine the rational-fraction and hyperbola coefficients. A more accurate representation for the magnetising characteristic is also proposed based on cubic spline interpolation.

2. MAGNETISING CHARACTERISTIC MODELS

The mathematical equations of the above different models are described in this section. Their adequacy to represent the magnetising characteristics of the various magnetic branches of a multilimb transformer is analysed based on the experimental saturation curves of a five-limb transformer given in reference [7].

The influence of these approximations upon the harmonic solution of an electric power system has been investigated using the 345 KV Jaguara-Taquaril test system [8]. Harmonic steady state simulation studies were carried out with the Harmonic Domain (HDA) program [9] with the three-phase bank of single phase transformers, using the main branch saturation curve given in [7].

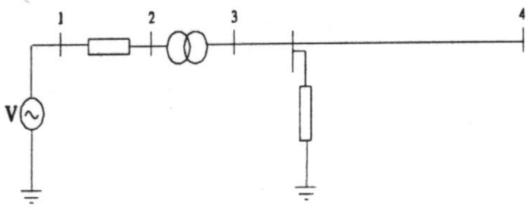


Figure 1. Test system

The Jaguara-Taquaril consists of a power plant feeding an unloaded 345 KV transmission line (resonance between the 6th and 7th harmonic frequency) through a step-up transformer, with shunt inductors connected on the secondary side as illustrated in Figure 1 (parameters described in [8]).

2.1. Piecewise linear approximations

Fourteen cases have been simulated for each of the magnetising characteristic approximations illustrated in Figure 2, i.e. two, three (curve 1) and four piecewise linear approximations. Each case has been obtained by varying the voltage source from 0.94 (pu) to 1.07 (pu). The HDA simulations included frequencies up to the 7th harmonic, and convergence checks have been restricted to voltages using a tolerance of 0.0001 (pu). Two additional three-piecewise linear approximations have been considered, curves 2 and 3 in Figure 2, in order to complement the sensitivity analysis.

The seventh harmonic voltage magnitude at

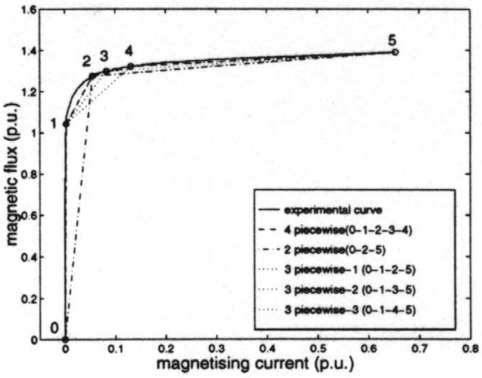
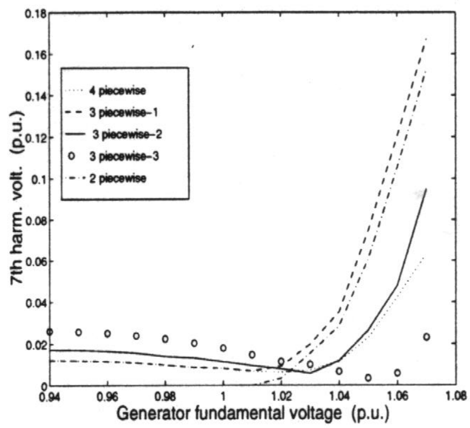
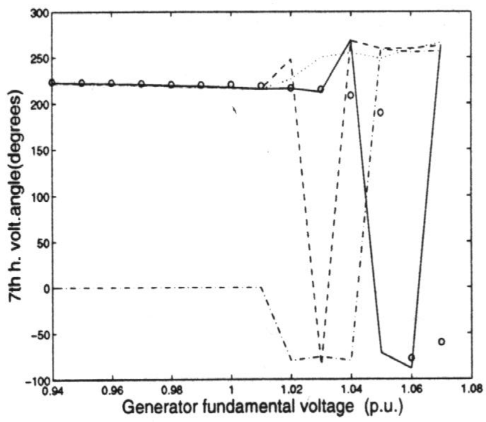


Figure 2. Piecewise linear approximations

busbar 3 are illustrated in Figure 3 (a). For applied voltages below 1.01 pu, i.e.when the transformer is not into deep saturation, with two piecewise linear approximation no harmonic distortion is produced. Above this operation point, the seventh harmonic voltages show significant discrepancies. Regarding the phase angle illustrated in Figure 3(b), abrupt variations occur for the two and three piecewise linear



(a)



(b)

Figure 3. Seventh harmonic voltages

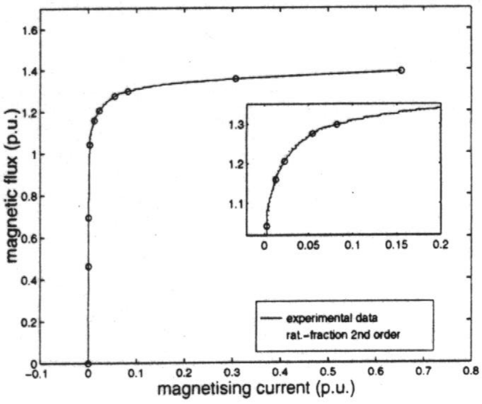
approximations, while a more steady value is observed for the four piecewise linear approximation. The differences clearly indicate the inadequacy of the two and three piecewise linear approximations.

2.2. Approximations by rational-fraction

The second-order rational-fraction interpolation approach proposed by Widger [2] is able to reproduce very accurately certain magnetising characteristics. The drawback of this approximation, given by the following equation

$$i_k = \frac{a_0 + a_1\phi_k + a_2\phi_k^2}{1 + b_1\phi_k + b_2\phi_k^2} \quad (k=1,\ldots,n) \qquad (1)$$

is the difficulty of determining its coefficients. The complexity has prevented this approximation from gaining a wide spread acceptance.



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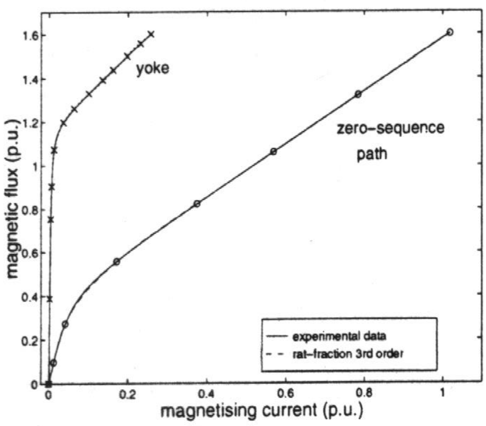


(b)

Figure 4. Rational-fraction interpolating curves

A simple mathematical approach, based on SVD, has been proposed to determine the coefficients of rational-fraction equations for frequency domain curve fitting problems [10] [11]. This approach has been adopted in this work, and its efficiency for magnetising characteristic approximations investigated.

Simulation results: The coefficients obtained for the main branch magnetising characteristic, yielded an interpolating curve that closely matches the experimental curve, as illustrated in Figure 4 (a), with an enlarged view of the knee region. These coefficients are given in Table 4.1 of [13].

The coefficients determined for the yoke and zero-sequence path magnetising characteristics could not produce accurate interpolating curves. Several simulations have been carried out varying the number of given points from the tabulated data without success. A good agreement is only obtained by increasing the rational-fraction order up to the third as illustrated in Figure 4 (b).

2.3. Approximations by hyperbola

The hyperbola equation [1][3] is given by

$$\varepsilon\,\phi_k = (m_1 i_k + b_1 - \phi_k)\,(m_2 i_k + b_2 - \phi_k)\; - b_1 \qquad (2)$$

In principle, all the coefficients can be approximately determined from the experimental curve, except the correction term @ that modulates the sharpness of the knee region of the magnetising characteristics. Nonetheless, an adequate determination of these parameters is not an easy task if an accurate representation of the whole curve is required, particularly those with a smooth knee region. The correction term @ itself is not sufficient to compensate for the approximations made when defining the other coefficients and if so, it still has the inconvenience of being determined by trial-and-error. Based on the method used to determine the rational-fraction coefficients, a similar method has been derived here to determine the hyperbola coefficients.

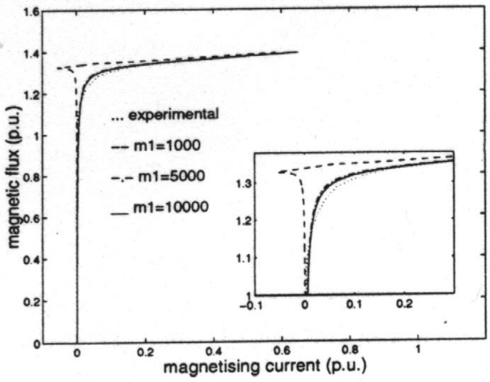
Considering that hysteresis is negligible, b_1 is equal to zero. Rewriting equation (2) as follows

$$(m_1 i_k^2 - i_k\,\phi)\, m_2 + (m_1 i_k - \phi_k)\, b2 - \phi_k\varepsilon = m_1 i_k \phi_k$$

and assuming that m_1 is known, the problem of determining the coefficients reduces to the solution of a set of over determined linear equations.

The weakness of this approach when applied to the hyperbola problem is the fact that m_1 in fact is not known. Nonetheless, as m_1 represents the slope of the unsaturated region, it can easily be approximated from the experimental data and by solving the equations in the least square sense, the other coefficients can be determined in such way that the inaccuracy in m_1 is compensated for.

Simulation results: The approach to determine the hyperbola coefficients is based on the assumption that m_l is given, and thus three values of m_l have been considered: 1000, 5000 and 10000, for the main branch magnetising characteristic, and the results are shown in Figure 5 (a).



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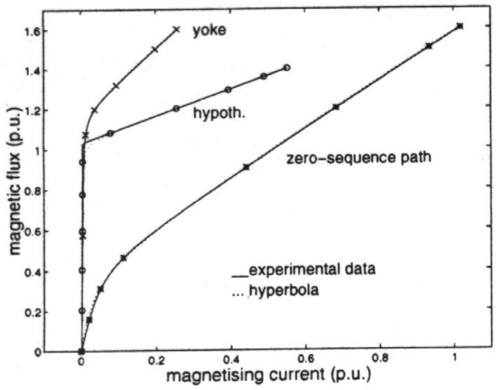


(b)

Figure 5. Hyperbola approximations

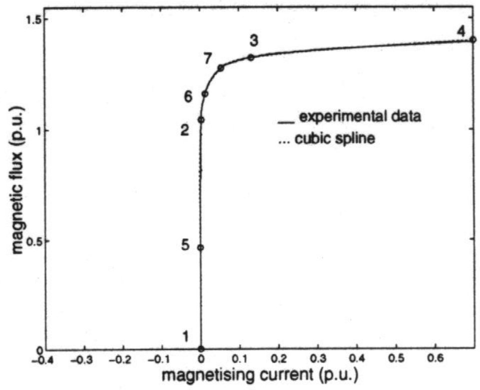
Contrary to what had been expected, with m_l equal to 1000 a very poor agreement is obtained, notably on the knee region. A very close match is obtained with m_l equal to 5000. However, if m_l is increased above this value, the accuracy gain is quite small, as can be seen from the result obtained with m_l equal to 10000. Only a small discrepancy is observed on the knee region for the best approximation.

With m_l equal to 10000 simulations have been carried out for the yoke and zero-sequence path magnetising characteristics and the results are illustrated in Figure 5 (b). Excellent agreements between interpolating and experimental curves have been obtained. The hyperbola coefficients are given in Table 4.2 of [13].

2.4. Cubic spline interpolation

Due to the simplicity of their fundamental properties, flexibility, and the availability of well-tested programs for their computation, spline functions are finding applications in an increasing number of numerical methods. They yield smooth interpolating curves which are less susceptible to large oscillations between tabulated points than high degree polynomials. Among them, cubic splines are the most commonly used and their theoretical details are well known (see [12]).

Simulation results: The cubic spline interpolating curves for the main, yoke and zero-sequence path magnetising characteristics, illustrated in Figure 6 (a) and (b), respectively, show a very close agreement between experimental and interpolating curves.



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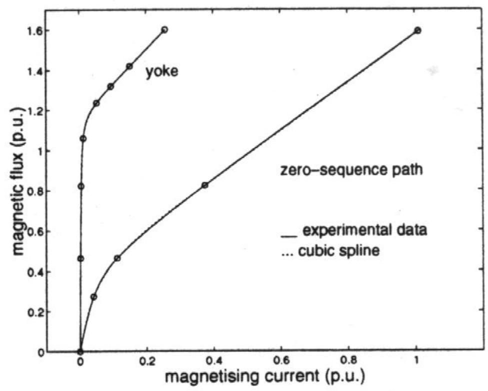

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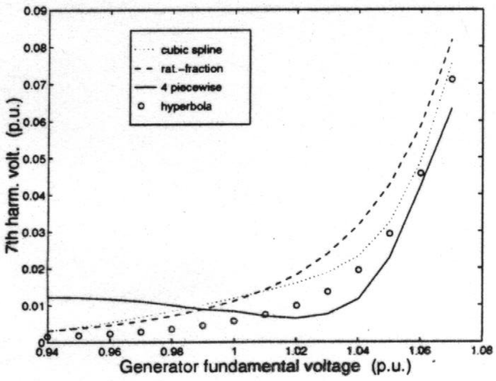
Figure 6. Natural cubic spline interpolating curves

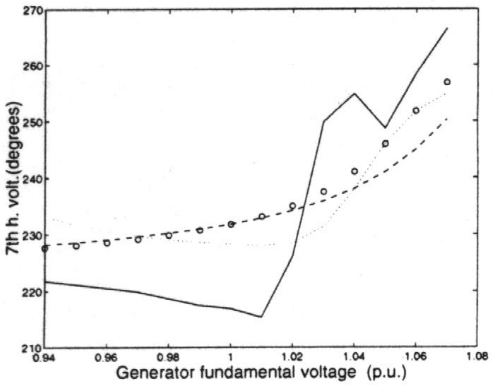
3. COMPARISON OF HARMONIC SOLUTIONS FOR DIFFERENT MAGNETISING CHARACTERISTIC MODELS

The harmonic solution of the Jaguara-Taquaril test system using different magnetising characteristic models is investigated in a similar way to that described in section 2.1. The 7th harmonic voltages at busbar 3, magnitude and phase angle, are shown in Figures 7 (a) and 7 (b), respectively.

The simulation results indicate a reasonably close harmonic solution for all saturation curve approximations. For the cases where the generator voltage is below 1.04 pu, the hyperbola and four piece wise linear approximations yield harmonic solutions slightly different from the others. For these cases, the transformer fundamental voltages are around the knee region and therefore, the differences are due to the poorer agreement of these two approximations with the knee region of the experimental curve.



(a)



(b)

Figure 7. Seventh harmonic voltages

4. CONCLUSIONS

For a power transformer with smooth knee region magnetising characteristic, an accurate reproduction of the entire curve is essential for a rigorous steady-state harmonic analysis of power systems with resonance problems. For this type of magnetising characteristic, two and three piecewise linear approximations can be rather inadequate, yielding results with a large margin of error.

The approach based on SVD technique to determine the rational-fraction and hyperbola coefficients is extremely simple and efficient. About ten points of the experimental curve, evenly distributed, are sufficient to determine excellent interpolating curves. However, for the rational-fraction the order of the numerator and denominator equation must be known, while for the hyperbola the slope of the unsaturated region must be given. The hyperbola yields interpolating curves slightly less accurate than rational-fraction approximations for some magnetising characteristics.

The cubic spline is a very simple, efficient and extremely accurate way to reproduce any type of magnetising characteristic. It can be extended to represent hysteresis and is suitable for harmonic domain algorithms based on convolutions.

The choice of the mathematical approximations used to represent the transformer magnetising characteristics is therefore, a matter of computational compatibility and availability of the relevant softwares.

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