Apparent Power: A Practical Approach to its Resolution

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Abstract – When the apparent power S, is correctly defined its square S^2 is in a nearly linear relation with the feeder power losses. Deviation from this rule gives a wrong figure for the power factor. Both the Apparent VA and the Arithmetic VA have this drawback, morever, even for balanced but nonsinusoidal loads the expression $\sqrt{3}V_{\ell\ell}I$ may not be accurate. It is shown that the only apparent power definition that holds this property for all possible conditions – balanced, unbalanced, sinusoidal or nonsinusoidal – is the effective apparent power S_e, suggested by F. Buchholz and explained by W. Goodhue. A practical resolution of S_e in active and nonactive powers is detailed. It is suggested to separate the 50 Hz or 60 Hz powers from the other components and to join all the nonactive powers in one separate entity.

Key Words – power Definitions. Harmonics. Power Quality

1. INTRODUCTION

It was shown in part I that parasitic oscillations of energy are a common feature for all four known nonactive power forms. It was concluded that for routine energy and power metering purpose it is practical to lump all the nonactive powers in one quantity with the exclusion of the fundamental reactive power.

Among several apparent power attributes, none is more consequential for deciding on a fair and wholesome definition, than the nearly linear relation between the feeder losses, ΔP and and the apparent power squared. It was shown that this is a "go-no-go" gage[1] that helps find if the apparent power definition is acceptable or not.

2. SINGLE-PHASE NONSINUSOIDAL SITUATIONS:

For steady-state conditions a nonsinusoidal instantaneous voltage or current has two distinctive components. The power system frequency component v_1 and i_1, and the remaining term v_H and i_H that contains all the harmonics, integers or noninteger as well as dc:

$$v_1 = \sqrt{2}V_1 \sin(\omega t - \alpha_1)$$

$$i_1 = \sqrt{2}I_1 \sin(\omega t - \beta_1)$$

$$v_H = \sqrt{2}\sum_{h\neq 1} V_h \sin(\omega t - \alpha_h)$$

$$i_H = \sqrt{2}\sum_{h\neq 1} I_h \sin(\omega t - \beta_h)$$

The corresponding rms values squared are:

$$V^2 = V_1^2 + V_H^2 \quad ; \qquad I^2 = I_1^2 + I_H^2 \qquad (1)$$

where:

$$V_H^2 = \sum_{h\neq 1} V_h^2 \quad ; \qquad I_H^2 = \sum_{h\neq 1} I_h^2 \qquad (2)$$

The overall **Apparent Power** squared is:

$$S^2 = (VI)^2 = (V_1^2 + V_H^2)(I_1^2 + I_H^2)$$

$$= (V_1 I_1)^2 + (V_1 I_H)^2 + (V_H I_1)^2 + (V_H I_H)^2$$

or

$$S^2 = S_1^2 + S_N^2 \qquad (3)$$

The first term is the **Fundamental Apparent Power (VA):**

$$S_1 = V_1 I_1 \qquad (4)$$

Electric utilities generate electric energy characterized by a sinusoidal 50 Hz or 60 Hz electromagnetic field. This energy is a product that is transmitted, distributed and sold to end-users. The consumers expect stable and symmetrical sinusoidal voltages. It is therefore logical to measure fundamental frequency powers separately from the rest of components that can be viewed as parasitic energies[2,3]. In single-phase systems the fundamental apparent power has two components:

Fundamental Active Power (W):

$$P_1 = V_1 I_1 \cos\theta_1 \qquad (5)$$

and **Fundamental Reactive Power (var):**

$$Q_1 = V_1 I_1 \sin\theta_1 \tag{6}$$

with the **Displacement Power Factor:**

$$PF_1 = \cos\theta_1 \tag{7}$$

The term S_N in (3) is the **Nonfundamental Apparent Power (VA)** that is resolved in three terms:

$$S_N^2 = D_I^2 + D_V^2 + S_H^2 \tag{8}$$

where:

$D_I = V_1 I_H$ is the **Current Distortion Power (vad? VA? var?)**
$D_V = V_H I_1$ is the **Voltage Distortion Power (vad? VA? var?)**
$S_H = V_H I_H$ is the **Harmonic Apparent Power (VA? vah?)**

The harmonic apparent power in turn can be divided in three components:

$$S_H^2 = P_H^2 + Q_H^2 + D_H^2$$

Harmonic Active Power (W):

$$P_H = \sqrt{2}\sum_{h\neq 1} V_h I_h \cos\theta_h \tag{9}$$

Harmonic Reactive Power (var):

$$Q_H = \sqrt{2}\sum_{h\neq 1} V_h I_h \sin\theta_h \tag{10}$$

and **Harmonic Distortion Power (vad? var?)**

$$D_H = \sqrt{S_H^2 - P_H^2 - Q_H^2} = \sqrt{\sum_{\substack{m\neq n\\ m,n\neq 1}} (V_m I_n)^2} \tag{11}$$

All the nonactive powers can be joined in one term, the **Nonfundamental Nonactive Power (var?)**:

$$N = \sqrt{S_N^2 - P_H^2} = \sqrt{D_I^2 + D_V^2 + Q_H^2 + D_H^2} \tag{12}$$

In summary, the powers that will convey reasonable information on energy transfer and power quality are:

S	S_1	P_1	S_N	N	P_H

Using the definitions of **Total Harmonic Distortion of Voltage and Current**:

$$THD_V = \frac{V_H}{V_1} \quad ; \qquad THD_I = \frac{I_H}{I_1} \tag{13}$$

in expression (3) gives for the nonfundamental apparent power:

$$S_N = S_1\sqrt{THD_I^2 + THD_V^2 + (THD_I THD_V)^2} \tag{14}$$

In practical power systems with $THD_V < 0.05$, and $THD_I > 0.38$ it is possible to replace (14) with a simple expression:

$$S_N \approx (THD_I) S_1 \tag{15}$$

When $0.05 < THD_I < 0.35$ accurate computations can rely on

$$S_N = S_1\sqrt{THD_I^2 + THD_V^2}$$

The **Power Factor** in nonsinusoidal systems is:

$$\begin{aligned} PF &= \frac{P}{S} = \frac{P_1 + P_H}{\sqrt{S_1^2 + S_N^2}} \\ &= \frac{(P_1/S_1)[1 + (P_H/P_1)]}{\sqrt{1 + (S_N/S_1)^2}} \\ &= \frac{[1 + (P_H/P_1)]PF_1}{\sqrt{1 + THD_I^2 + THD_V^2 + (THD_I\ THD_V)^2}} \\ &\approx \frac{1}{\sqrt{1 + THD_I^2}} PF_1 \end{aligned} \tag{16}$$

3. SINUSOIDAL AND UNBALANCED THREE-PHASE SYSTEMS

In the past decades a host of apparent power definitions for polyphase systems were proposed by different schools of thought. Many of the suggested definitions lack the attributes of S, that were explained in part I. One apparent power definition however, gained the confidence of a significant number of researchers. It is known under the names of Effective, or Equivalent, or System's Apparent Power S_e. It was suggested by F. Buchholz[3] in 1922 and explained in 1933 by W. Goodhue[4]. This definition of S_e is based on an equivalent balanced circuit that has exactly the same power losses as the actual unbalanced circuit. From Fig.1 results that the balance of power loss in the actual unbalanced system is:

$$\begin{aligned} \Delta P_{actual} &= r(I_a^2 + I_b^2 + I_c^2 + I_n^2) \\ &+ \frac{V_a^2 + V_b^2 + V_c^2}{R} + \frac{V_{ab}^2 + V_{bc}^2 + V_{ca}^2}{3R} \end{aligned} \tag{17}$$

The power loss in the fictitious balanced system is:

$$\Delta P_e = 3rI_e^2 + 3\frac{V_e^2}{R} + \frac{9V_e^2}{3R} \qquad (18)$$

where V_e is the line-to-neutral equivalent voltage and I_e is the equivalent line current.
The equivalence of the two circuits is expressed in the power loss equality, $\Delta P_e = \Delta P_{actual}$. This equation gives the **equivalent current and voltage** for a 4-wire system:

$$I_e = \sqrt{\frac{I_a^2 + I_b^2 + I_c^2 + I_n^2}{3}} = \sqrt{I_+^2 + I_-^2 + 4I_0^2} \qquad (19)$$

and

$$V_e = \sqrt{\frac{1}{18}[3(V_a^2 + V_b^2 + V_c^2) + V_{ab}^2 + V_{bc}^2 + V_{ca}^2]} = \sqrt{V_+^2 + V_-^2 + \frac{V_0^2}{2}} \qquad (20)$$

In the same manner one finds the equivalent voltage and current for a 3-wire system:

$$3rI_e^2 + \frac{9V_e^2}{3R} = r(I_a^2 + I_b^2 + I_c^2) + \frac{V_{ab}^2 + V_{bc}^2 + V_{ca}^2}{3R}$$

and from here results:

$$I_e = \sqrt{\frac{I_a^2 + I_b^2 + I_c^2}{3}} = \sqrt{I_+^2 + I_-^2} \qquad (21)$$

and

$$V_e = \sqrt{\frac{V_{ab}^2 + V_{bc}^2 + V_{ca}^2}{9}} = \sqrt{V_+^2 + V_-^2} \qquad (22)$$

The **effective apparent power is:**

$$S_e = 3V_eI_e \qquad (23)$$

and the effective power factor:

$$PF_e = P/S_e \qquad (24)$$

In the above equations V_+, V_-, V_0, I_+, I_- and I_0 are the symmetrical components of voltage and current.

An important topic for future debates stems from the value chosen for the effective resistance of the zero-sequence current return path $r_n = \rho r$ and shunt resistances R. In the equations of ΔP_e and ΔP_{actual} it was assumed that phase conductors and neutral conductors have the same resistance r, i.e $r = r_n$, $\rho = 1$. In reality these resistances are not necessarily equal. Skin and proximity effects cause a significant frequency dependence $r = r(h)$, (h=harmonic order.) A large scale survey will be needed to establish a satisfactory universal values for the coefficient ρ of the return path for the zero-sequence currents. In the second group fictitious resistances R represent "fixed" losses, i.e. magnetic core losses in transformers and dielectric losses in high voltage cables. In this model it was assumed that the fixed losses due to equipment connected line-to-neutral are equal to the losses caused by components connected line-to- line. Field measurements prove that $V_e \approx V$, hence the quest over R and $3R$ has academic significance only.

The IEEE Std.100 endorses two well known apparent power definitions[6]: one is the Vector VA or Vector Apparent Power:

$$S_V = \sqrt{(P_a + P_b + P_c)^2 + (Q_a + Q_b + Q_c)^2} \qquad (25)$$

where:

$$P_a = V_aI_a\cos\theta_a; \quad Q_a = V_aI_a\sin\theta_a$$

$$P_b = V_bI_b\cos\theta_b; \quad Q_b = V_bI_b\sin\theta_b$$

$$P_c = V_cI_c\cos\theta_c; \quad Q_c = V_cI_c\sin\theta_c \qquad (26)$$

$$P = P_a + P_b + P_c \qquad Q = Q_a + Q_b + Q_c \qquad (27)$$

and the Vector VA Power Factor:

$$PF_V = P/S_V \qquad (28)$$

The second is the Arithmetic VA or Arithmetic Apparent Power:

$$S_A = S_a + S_b + S_c \qquad (29)$$

where

$$S_a = V_aI_a \qquad S_b = V_bI_b \qquad S_c = V_cI_c \qquad (30)$$

and the arithmetic VA power factor:

$$PF_A = P/S_A \qquad (31)$$

When the system is balanced:

$$V_a = V_b = V_c = V_{\ell n} = V_e$$

$$I_a = I_b = I_c = I_\ell; \qquad I_n = 0$$

and

$$S_V = S_A = S_e$$

When the system is unbalanced:

$$S_V \le S_A \le S_e$$

and

$$PF_V \ge PF_A \ge PF_e$$

The effective power S_e satisfies the requirement of linearity ΔP versus S_e^2 based on the very model sustained by (17) and (18). The vector and the arithmetic apparent powers do not fulfill this condition[6]

4. NONSINUSOIDAL BALANCED THREE-PHASE SYSTEMS

The line-to-neutral voltages are:

$$v_a = \sqrt{2}V_1 \sin\omega t + \sqrt{2}\sum_{h\neq 1} V_h \sin(h\omega t + \alpha_h)$$

$$v_b = \sqrt{2}V_1 \sin(\omega t - 120^o) + \sqrt{2}\sum_{h\neq 1} V_h \sin(h\omega t + \alpha_h - 120^o h)$$

$$v_c = \sqrt{2}V_1 \sin(\omega t + 120^o) + \sqrt{2}\sum_{h\neq 1} V_h \sin(h\omega t + \alpha_h + 120^o h)$$

The line currents have similar expressions:

$$i_a = \sqrt{2}I_1 \sin\omega t + \sqrt{2}\sum_{h\neq 1} I_h \sin(h\omega t + \beta_h)$$

$$i_b = \sqrt{2}I_1 \sin(\omega t - 120^o) + \sqrt{2}\sum_{h\neq 1} I_h \sin(h\omega t + \beta_h - 120^o h)$$

$$i_c = \sqrt{2}I_1 \sin(\omega t + 120^o) + \sqrt{2}\sum_{h\neq 1} I_h \sin(h\omega t + \beta_h + 120^o h)$$

One will notice two significant characteristics for these voltages and currents:

1. The line-to-line voltages do not contain zero-sequence harmonics ($h = 0, 3, 6, 9, \ldots$) for example:

$$v_{ab} = v_a - v_b = \sqrt{3}\sqrt{2}V_1 \sin(\omega t + 30^o)$$

$$+\sqrt{3}\sqrt{2}\sum_{h\neq 0,3,6,9,\ldots..} V_h \sin(h\omega t + \alpha_h + 30^o h)$$

This means that

$$V_{\ell n} = \sqrt{\sum_h V_h^2} \ge \frac{V_{\ell\ell}}{\sqrt{3}} = \sqrt{\sum_{h\neq 0,3,6,9,\ldots.} V_h^2}$$

2. The neutral current is not nil. Since the zero-sequence current harmonics add-up in the neutral conductor:

$$i_n = i_a + i_b + i_c = 3\sqrt{2}\sum_{h=0,3,6,9,\ldots.} I_h \sin(h\omega t + \beta_h)$$

These two properties lead to two important conclusions for 4-wire systems:

a. The basic apparent power definitions $3V_{\ell n}I$ and $\sqrt{3}V_{\ell\ell}I$ do not give identical results:

$$3V_{\ell n}I_\ell > \sqrt{3}V_{\ell\ell}I_\ell$$

The use of definition $\sqrt{3}V_{\ell\ell}I$ should be avoided when

$$\frac{\sqrt{\sum_{h=0,3,6,9,\ldots.} V_h^2}}{V_1} > 0.10$$

b. There are additional power losses caused by the neutral current. These losses are not reflected in the definitions $3V_{\ell n}I$ and $\sqrt{3}V_{\ell\ell}I$ and will lead to a wrong PF value.

These observations lead to the conclusion that for three-phase systems with nonsinusoidal wave forms the effective apparent power S_e and its components offer an improved set of definitions that help evaluate the power flow conditions. The resolution of S_e for three-phase systems is detailed in the next section.

5. THREE-PHASE UNBALANCED AND NONSINUSOIDAL SYSTEMS

Expanding the approach presented in sections 2. and 3. one will find the equivalent currents and voltages:

$$I_e = \sqrt{I_{e1}^2 + I_{eH}^2} \quad (32)$$

$$V_e = \sqrt{V_{e1}^2 + V_{eH}^2} \quad (33)$$

where for a 4-wire system:

$$I_{e1}^2 = \frac{I_{a1}^2 + I_{b1}^2 + I_{c1}^2 + I_{n1}^2}{3} \quad (34)$$

$$I_{eH}^2 = \frac{I_{aH}^2 + I_{bH}^2 + I_{cH}^2 + I_{nH}^2}{3} \quad (35)$$

$$V_{e1} = \sqrt{\frac{1}{18}[3(V_{a1}^2 + V_{b1}^2 + V_{c1}^2) + V_{ab1}^2 + V_{bc1}^2 + V_{ca1}^2]}$$

(36)

$$V_{eH} = \sqrt{\frac{1}{18}[3(V_{aH}^2 + V_{bH}^2 + V_{cH}^2) + V_{abH}^2 + V_{bcH}^2 + V_{caH}^2]}$$

(37)

For 3-wire systems $I_{n1} = I_{nH} = 0$ and from (21) results:

$$V_{e1} = \sqrt{\frac{V_{ab1}^2 + V_{bc1}^2 + V_{ca1}^2}{9}} \tag{38}$$

$$V_{eH} = \sqrt{\frac{V_{abH}^2 + V_{bcH}^2 + V_{caH}^2}{9}} \tag{39}$$

The resolution of $S_e = 3V_e I_e$ is implemented in the manner shown in section 2.:

$$S_e^2 = S_{e1}^2 + S_{eN}^2 \tag{40}$$

where

$$S_{e1} = 3V_{e1} I_{e1} \tag{41}$$

and

$$S_{eN}^2 = D_{eI}^2 + D_{eV}^2 + S_{eH}^2 \tag{42}$$

The current distortion power, voltage distortion power and harmonic apparent power are:

$$\begin{aligned} D_{eI} &= 3V_{eI} I_{eH} \\ D_{eV} &= 3V_{eH} I_{e1} \\ S_{eH} &= 3V_{eH} I_{eH} \end{aligned} \tag{43}$$

The most important component of the fundamental apparent power is the positive-sequence fundamental apparent power S_1^+. In turn S_1^+ is divided in positive-sequence fundamental active and reactive powers:

$$P_1^+ = 3V_1^+ I_1^+ \cos\theta_1^+; \quad Q_1^+ = 3V_1^+ I_1^+ \sin\theta_1^+ \tag{44}$$

They form a fundamental positive-sequence power factor:

$$PF_1^+ = P_1^+ / S_1^+$$

This utilization factor ought not be confused with the displacement factors $\cos\theta_{a1}$, $\cos\theta_{b1}$ and $\cos\theta_{a1}$. These three factors are not necessarily equals.

The load power factor or the system's power factor is:

$$PF = (P_1 + P_H)/S_e$$

The remaining component of S_{e1}

$$S_{u1} = \sqrt{S_{e1}^2 - (S_{e1}^+)^2} \tag{45}$$

contains the negative and zero-sequence fundamental powers. The ratio S_{u1}/S_1 is an index that can help estimate the degree of linear or nonlinear load unbalance.

The nonactive powers can be gathered in one quantity, the **nonfundamental nonactive power**:

$$N_e = \sqrt{S_{eN}^2 - P_H^2}$$

Defining **Equivalent Total Harmonic Distortions:**

$$THD_{eV} = \frac{V_{eH}}{V_1} \quad ; \qquad THD_{eI} = \frac{I_{eH}}{I_1} \tag{46}$$

helps to obtain for the nonfundamental apparent power S_{eN} an expression identical with (15):

$$\begin{aligned} S_{eN} &= S_{e1}\sqrt{THD_{eI}^2 + THD_{eV}^2 + (THD_{eI} THD_{eV})^2} \\ &\approx S_{e1}\sqrt{THD_{eI}^2 + THD_{eV}^2} \approx S_{e1}(THD_{eI}) \end{aligned} \tag{47}$$

The IEEE Std. 100 expands the definitions of the vector and arithmetic apparent powers to nonsinusoidal systems too. These expressions stem from Budeanu's definitions for single-phase nonsinusoidal systems, applied individually to each phase :

$$P_a = \sum_h V_{ah} I_{ah} \cos\theta_{ah}; \quad Q_a = \sum_h V_{ah} I_{ah} \sin\theta_{ah}$$

$$P_b = \sum_h V_{bh} I_{bh} \cos\theta_{bh}; \quad Q_b = \sum_h V_{bh} I_{bh} \sin\theta_{bh}$$

$$P_c = \sum_h V_{ch} I_{ch} \cos\theta_{ch}; \quad Q_c = \sum_h V_{ch} I_{ch} \sin\theta_{ch}$$

$$D_a = \sqrt{S_a^2 - P_a^2 - Q_a^2}$$

$$D_b = \sqrt{S_b^2 - P_b^2 - Q_b^2}$$

$$D_c = \sqrt{S_c^2 - P_c^2 - Q_c^2}$$

with

$$S_a = V_a I_a = \sqrt{P_a^2 + Q_a^2 + D_a^2}$$

$$S_b = V_b I_b = \sqrt{P_b^2 + Q_b^2 + D_b^2}$$

$$S_c = V_c I_c = \sqrt{P_c^2 + Q_c^2 + D_c^2}$$

The vector apparent power and power factor are:

$$S_V = \sqrt{P^2 + Q^2 + D^2} \quad ; \qquad PF_V = P/S_V \tag{48}$$

where

$$P = P_a + P_b + P_c$$

$$Q = Q_a + Q_b + Q_c$$

and

$$D = \sqrt{S_V^2 - P^2 - Q^2}$$

The arithmetic apparent power and power factor are:

$$S_A = S_a + S_b + S_c \quad ; \qquad PF_V = P/S_A \tag{49}$$

In the general case of nonsinusoidal and unbalance system the significant powers are the following:

S_e	S_1^+	P_1^+	S_{Ne}	N_e	S_u	P_H

6. CONCLUSIONS

The Buchholz - Goodhue apparent power, S_e, is the only known apparent power that leads to correct power factor computation not only for unbalanced sinusoidal and nonsinusoidal systems, but also for balanced nonsinusoidal systems. An incorrect apparent power value translates in an incorrect power factor. This result has two repercussions: First is the incorrect VA evaluation. Readers are reminded the incidents flagged in the engineering literature in the 1980's when third harmonic producing equipment was unexpectedly overloading and overheating neutral conductors and transformers. This problem was due to misunderstanding both the power loss distribution within equipment and the fact that apparent power for nonsinusoidal conditions has a different structure than for sinusoidal. The second consequence is the inequitable penalty of end-users that degrade service reliability.

The S_e division in active and nonactive components is based on the separation of 50 Hz or 60 Hz powers from the rest. This approach leads to two distinct apparent powers, the fundamental and nonfundamental apparent powers, S_{e1} and S_{eN}. The fundamental apparent power is by far dominated by the positive sequence powers, i.e the fundamental frequency positive-sequence apparent, active and reactive powers S_1^+, P_1^+ and Q_1^+. The fundamental unbalance apparent power $S_{u1} = \sqrt{S_1^2 - (S_1^+)^2}$ contains the nonpositive-sequence powers, i.e. the negative and zero-sequence powers. These powers are produced by the unbalanced loads by converting a small part of the positive-sequence powers into nonpositive-sequence powers. The ratio S_{u1}/S_1^+, helps evaluate load unbalance severity.

The impact of an harmonic polluting load may be approximated with the help of the nonfundamental nonactive power N_e, that gathers under one umbrella all the nonactive powers but fundamental. In many applications were the current is severely distorted, $N_e \approx S_{eN}$ and the equivalent total harmonic distortion of the current is:

$$THD_{eI} \approx \frac{N_e}{S_{e1}} \tag{50}$$

This expression could become in the future the key to weighing the harmonic pollution caused by end-users. Since the voltage waveform remains nearly sinusoidal (50) indicates that the amount of pollution can be estimated from the product $THD_{eI}S_{e1}$ or just the current I_{eH}.

7. REFERENCES

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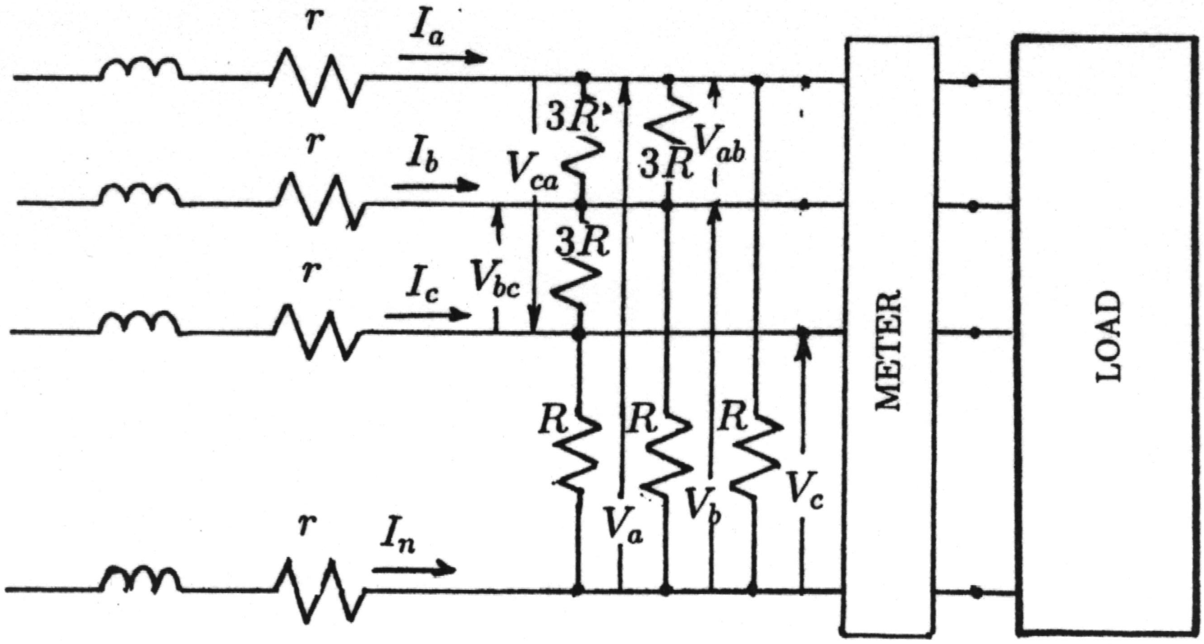
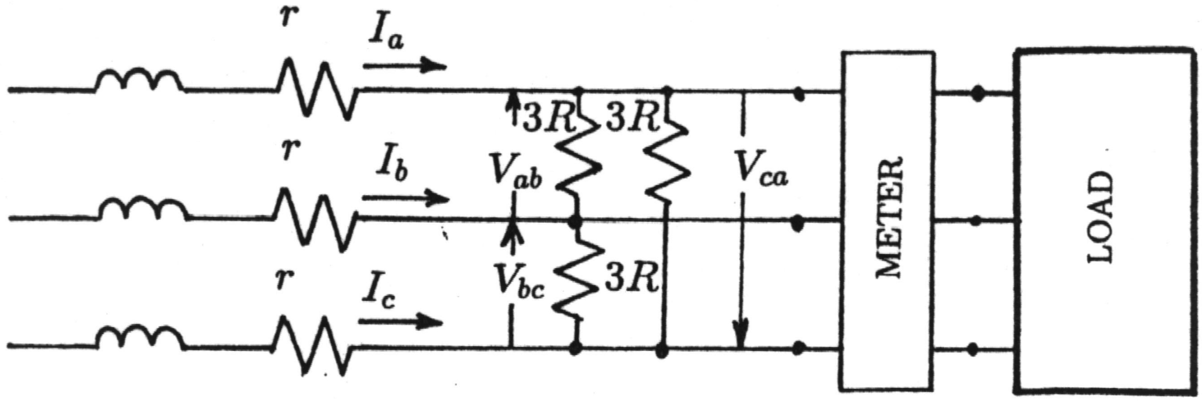


(a)



(b)

Fig.1 Unbalanced Three-Phase Circuits

(a) Four-Wire System
(b) Three-Wire System