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# Identification of Short-circuit and Induction Motor Starting Events on Power Systems Monitoring and Protection Using Instantaneous Space Vectors

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Abstract—This document presents the applications of recently defined quantities: the instantaneous complex power and the Buchholz-Goodhue apparent power, which are obtained using instantaneous space vectors. They are applied in a conceptive way to the analysis of an induction motor starting and a short circuit on a short-distance distribution line to show how those quantities can be used for power systems monitoring and protection purposes. A new electrical circuit concept about instantaneous impedance is also presented, along with the method for obtaining it on on-line and real time measurement conditions. Some other new power concepts are also introduced.

Index Terms— Buchholz-Goodhue, induction machines, instantaneous complex power, instantaneous space vectors, power concepts, power quality, power systems monitoring, power systems protection.

# I. INTRODUCTION

The use of instantaneous space vectors (or phasors) comes **I** originally from the transient analysis of ac machines mainly due to the works of Park and Clarke in the twenties [1]. In the sixties and seventies Hasse and Blaschke [10]-[2], presented a methodology to control these machines in a way known as vector control. Later it was demonstrated that those vectors were well suited to be applied to any threephase power system power flow analysis operating on sinusoidal or non-sinusoidal, balanced or unbalanced steadystate conditions (non-ideal power systems) and also for transient conditions [3], [6], [7], as an alternative way to write and represent the well known Akagi's theory of instantaneous reactive power [8], [9]. Recently it was demonstrated that instantaneous space vectors can also be applied to calculate power quality indices of non ideal three-phase power systems based on Buchholz-Goodhue quantities [4].

In this work it will be shown how far can go the application of instantaneous space vectors on power systems monitoring and analysis. It will be focused here some analysis results of an induction motor starting transient that can be used for monitoring power systems events and parameter identification of power lines and loads.

Starting of induction motors is one of the main causes of voltage sags in power systems. Sometimes it is mistaken as a three-phase short-circuit on power lines, causing malfunction of protective relays. To distinguish between three-phase short-circuits and induction motor starting we will use here a new method for power and current decomposition, introducing in this way new concepts for electrical circuits and power studies. We believe that the mathematical tools here applied expand our knowledge on analyzing power systems and that the main contribution of this paper is to show their application on power systems analysis, monitoring and protection, through the analyzed cases.

### II. BACKGROUND

On a three-phase power system the phase voltages  $v_a$ ,  $v_b$  and  $v_c$ , and the phase currents  $i_a$ ,  $i_b$  and  $i_c$  can be multiplied by the unity complex vectors  $a^0$ , a and  $a^2$  respectively, and added to obtain the following respective complex equations:

$$\widetilde{V} = \frac{2}{3} \left( v_a + a v_b + a^2 v_c \right) \tag{1}$$

$$\widetilde{I} = \frac{2}{3} \left( i_a + a i_b + a^2 i_c \right) \tag{2}$$

where  $a = e^{j\frac{2\pi}{3}}$ 

Equations (1) and (2) are written on a complex plane  $\alpha - \beta$  whose real axis  $\alpha$  coincides with the reference axis for phase a, because we multiplied the phase a variables by  $a^0$ , which is the unity vector that gives the direction of the real axis.

Such transformations are known as Clarke's transformations. The resultant vectors are called instantaneous space vectors and can be represented in polar form as

$$\widetilde{V} = |\widetilde{V}| e^{j\phi_V}, \ \widetilde{I} = |\widetilde{I}| e^{j\phi_I}$$
 (3)

These complex vectors may be multiplied to give the expression for the instantaneous complex power [5], [6],

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$$\widetilde{S} = \frac{3}{2} \left( \widetilde{V} \, \widetilde{I}^* \right) = \frac{3}{2} \left| \widetilde{V} \right| \left| \widetilde{I} \right| e^{j(\phi_V - \phi_I)}$$

$$= S \left( \cos \theta + j \sin \theta \right)$$

$$= p + j q$$
(4)

whose real and imaginary parts give respectively Akagi's real and imaginary powers for three-phase circuits without neutral conductor.

### III. INDUCTION MOTOR STATING ANALYSIS

The current instantaneous space vector trajectory during the transient starting period of an induction motor, obtained from the simulation of its dynamical model [11] (see the parameter values in Appendix B), is a growing spiral in a  $\alpha - \beta$  complex plane shown in Fig. 1.

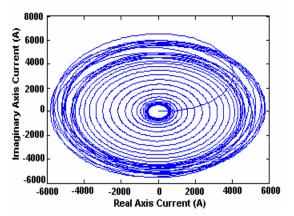


Fig.1—Current instantaneous space vector trajectory during an induction motor starting.

The amplitude of the current instantaneous space vector as well as its angular velocity are time varying quantities. Fig. 2 shows its angular velocity variation.

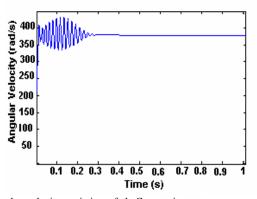


Fig. 2 Angular velocity variation of theCurrent instantaneous space vector during the induction motor starting.

A very interesting result can be obtained by drawing the current instantaneous space vector locus shown in Fig. 1 on a new coordinate system d-q, a synchronous coordinate system, which rotates at synchronous electrical angular velocity  $\omega_s$  imposed by the symmetrical three-phase voltage source. This operation will give the machine's circle diagram for current or its circle diagram for power. These

circles have their coordinates proportional to each other. Its is easy to see this proportionality if we consider that the conjugate of the instantaneous complex power given by (4) is

$$\widetilde{S}^* = \frac{3}{2}\widetilde{V}^*\widetilde{I} = \frac{3}{2}\sqrt{2}\dot{V}^* \exp(-j\omega_s t)\widetilde{I}$$
 (5)

since in this case  $\tilde{V}$  is a rotating vector that rotates at synchronous speed and with constant amplitude and it can be written as

$$\widetilde{V} = \sqrt{2} \, \dot{V} \exp(j \, \omega_{\rm s} \, t) \tag{6}$$

where

$$\dot{V} = V \exp(j \phi_{V0})$$

is the conventional phasor notation for steady-state circuit analysis.

Now if we take the instantaneous current vector in the new d-q rotating coordinate system

$$\widetilde{I}_{d-q} = \widetilde{I}_{\alpha-\beta} \exp(-j\omega_s t) \tag{7}$$

we can easily find that the conjugate of the instantaneous complex power given by (5) is proportional to the current instantaneous space vector represented on this coordinate system.

The instantaneous complex power graphic or power diagram of the machine is presented in Fig. 3.

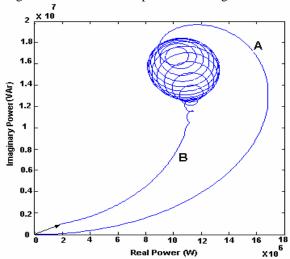


Fig. 3 Instantaneous complex power locus during an induction motor starting.

It is possible to separate from the total input real power, the copper losses, the mechanical power and electromagnetic power using new concepts of mathematical power representation (see Appendix A). This separation can be done on-line through the measurement of voltages and currents at the machine's terminals. Fig. 4 shows these powers delivered to the motor.

Considering this power decomposition (Fig. 4), a further analysis on the power diagram (Fig. 3) allows us to conclude that it consists of two parts:

- A- Period of electromagnetic power delivering, during which the magnetic rotating field is established and the rotor starts running (about 0,2 s);
- B- Period of electromechanical power developing, when the machine is accelerating after having its rotating field established (This part of the curve coincides

with the machine's circle diagram obtained for its steady-state operation, in function of the slip *s*).

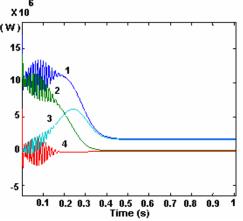


Fig. 4 Total real power input (curve 1), total copper losses (curve 2), mechanical power output (curve 3) and electromagnetic power (curve 4) during an induction motor starting.

This power separation allows better understanding of the nature of the complex power, its real and its imaginary part and useful applications as seen in the next sections.

### IV. POWER SYSTEM MONITORING AND PROTECTION

The importance of circle diagrams in power systems is well known by the electrical engineering people. If we consider that the machine is fed through a power distribution line we can have a mean to evaluate the line plus machine's parameters which can be done by digital relays and programming these relays to trip only when necessary.

### A. The machine fed by an infinite bus

Since we have the machine's circle diagram which correspond to the curve B of the diagram of Fig. 3 , for every point of this circle the machine is considered running at steady-state because the voltage instantaneous space vector has constant amplitude and constant angular velocity, and the current instantaneous space vector (although it has variable amplitude this variation is very slow within each cycle due to the mechanical time constant of the rotor and load) has also constant angular velocity. Then we can define an instantaneous equivalent impedance given by

$$\widetilde{Z}_{inst} = \frac{\widetilde{V}}{\widetilde{I}} = \frac{\left|\widetilde{V}\right| exp(j\omega_s t)}{\left|\widetilde{I}\right| exp(j\omega_s t)} = \frac{\sqrt{2} \, \widetilde{I}}{\left|\widetilde{I}\right|} \tag{8}$$

Only for the electromagnetic period (part A of the curve in Fig. 3) it is not true, because in this case the current instantaneous space vector has a great oscillation on its angular velocity as shown in Fig. 2.

For the steady-state part of the curve (part B), the instantaneous equivalent impedance varies due to the slip s variation.

This impedance variation can now be plotted as shown in Fig. 5.

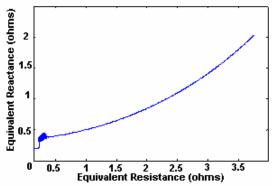


Fig. 5 The instantaneous equivalent impedance variation during the induction motor starting computed at its terminal.

For on-line monitoring of power systems we may plote this curve, and for protective purposes, using digital impedance relays, the impedance of the line plus the instantaneous impedance of the machine can be obtained in real time as seen in the next section.

# B. The machine fed by a short-distance transmission line.

In this case there is a voltage drop on the transmission line. Then the voltage at the machine's terminals presents a sag as shown in Fig. 6.

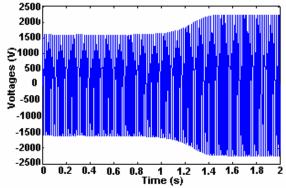


Fig. 6 Voltage sag at the machine's terminals during its starting.

In this case both voltage and current do not have their magnitudes constant. Then (8) becomes

$$\widetilde{Z}_{inst} = \frac{\widetilde{V}}{\widetilde{I}} = \frac{\left|\widetilde{V}\right| exp(j\omega_s t)}{\left|\widetilde{I}\right| exp(j\omega_s t)} = \frac{\left|\widetilde{V}\right|}{\left|\widetilde{I}\right|}$$
(9)

The locus for this instantaneous equivalent impedance is presented in Fig. 7. We can see that it is slightly lower than that one for the infinite bus case. Equation (9) may be useful for monitoring and protection purpose.

# C. Short circuit at the end of the transmission line.

In this case there is a very fast impedance variation, and only the line's parameters are present.

Only the impedance prior the short-circuit and the impedance after the short-circuit (the short-circuit conventional impedance) are of interest. This later can be calculated in real time using the new method of magnetic power separation presented in Appendix A.

# D. Digital impedance relay graphics

Fig. 7 shows protection areas (only for illustration) of a digital impedance relay, the instantaneous equivalent motor impedance variation (curve 1), the line plus motor instantaneous equivalent impedance variation (curve 2) and the line impedance variation for the case of a symmetrical three-phase short-circuit at the end of it (curve 3).

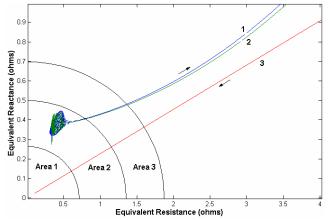


Fig. 7 Impedance diagram as seen by an impedance relay: curve 1 (infinite bus, only the machine's impedance), curve 2 (short-distance line plus machine's impedance), curve 3 (line impedance during a short-circuit).

This graph can give us a qualitative and quantitative information about the eletromagnetic event taken place. Since the process of stanting of induction machines is slower than short circuits, curves 1 and 2 have a slow time variation compared with curve 3 (time is implicit is that graph), and it moves upward (the instantaneous impedance incrisis because the equivalente motor impedance varies from its locked rotor value to its full-load value, in a velocity that depends on the motor's mechanincal time constant). Curve 3 moves downward (from a situation that includes the line parameter plus other loads prior the short-circuit to the line parameter at the distance which the short-circuit ocurs.

The time derivatives of those curves as well as their initial and final values can be used for programing the digital impedance relay.

### V. MACHINE'S PARAMETERS ESTIMATION

As the circle diagram can be obtained on-line and used to estimate the machine's parameters, we have here a new method to replace the old method of essays: the locked rotor essay and the no-load essay. But for a better precision on the computation of the locked rotor impedance we must separate the electromagnetic power. Then for s = 1 we can use the machine's equivalent circuit considering it at steady-state. To do so consider the method presented in Appendix A.

### VI. CONCLUSIONS

The following considerations can be extracted from the results presented on this paper:

All the figures shown here can be obtained on-line by registering voltages and currents after the starting of the studied power systems event.

For the purpose of analysis, those diagrams may be compared with some patterns obtained for several types of loads and events.

For the purpose of protection it is suggested that the line plus motor equivalent impedance can be calculated in real time and the relay must not trip for the case of induction motor starting, even when that impedance reaches its tripping region.

To avoid confusion between motors starting events and three-phase symmetrical short-circuits one more information can be used by the relay: the electromagnetic power flow through the relay and the time derivative of the instantaneous impedance variation.

Since, according to the induction machine theory, starting from the knowledge of machine's parameter one can draw its circle diagram, conversely, if we have the machine's circle diagram we may calculate its parameters. Then, this is a new method that can replace the old methods for parameter calculation: the blocked-rotor essay and the no-load essay.

It may be interesting for the power system's operator to know more information about the starting motors which causes disturbances like voltage sags. In this way the machine's HP may be estimated using its circle diagram.

For the industry engineer it may be useful to drawn the machines circle diagram and obtaining its parameters on-line for dimensioning the protection, conductors and power factor correction equipment.

The process for obtaining the resistive and inductive parts of a three-phase R-L balanced circuit during a transient, using instantaneous vectors, introduces new concepts not yet known from the electrical circuit theory.

# VII. APPENDIX A

To obtain the power separation on a three-phase balanced R-L load let us consider the expression of the instantaneous complex delivered to it, according to the instantaneous complex power theory [6], [7],

$$\widetilde{S} = \frac{3}{2} \left( R \left| \widetilde{I} \right|^2 + L \left| \widetilde{I} \right| \frac{d \left| \widetilde{I} \right|}{dt} + j \omega_I L \left| \widetilde{I} \right|^2 \right)$$
 (10)

The first term in the right hand side of this expression is the power dissipated by Joule's effect, the second term the magnetic power and the last term the imaginary power. The two first ones are real powers, and the time integral of the magnetic power is the magnetic energy stored on the inductances, given by

$$E_l = \frac{3}{2}L|\tilde{I}|^2 \tag{11}$$

Notice that [4]

$$\frac{3}{2}L|\tilde{I}|^2 = \frac{1}{2}L(i_a^2 + i_b^2 + i_c^2)$$
 (12)

where  $i_a$ ,  $i_b$  and  $i_c$  are the phase currents respectively of phases a, b and c of the three-phase circuit.

Comparing this expression of magnetic energy with the imaginary power we find that

$$E_l = \frac{Q}{2\omega_l} = \frac{3}{2}L\left|\tilde{I}\right|^2 \tag{13}$$

where

$$Q = |\widetilde{V}||\widetilde{I}|\sin(\theta) \tag{14}$$

is the imaginary power delivered to that load and  $\theta$  the angle between  $\tilde{V}$  and  $\tilde{I}$  .

Then (13) gives the following relation:

$$\left| \widetilde{V}_{Q} \right| = \left| \widetilde{V} \right| \sin \left( \theta \right) = 3 \, \omega_{I} \, L \left| \widetilde{I} \right| \tag{15}$$

where  $\tilde{V}_O$  is orthogonal to  $\tilde{I}$  .

Then we may write (15) in its phasor form as

$$\widetilde{V}_{Q} = j \, 3 \, \omega_{I} \, L \, \widetilde{I} \tag{16}$$

and we define the instantaneous reactance as

$$X = 3\omega_1 L \tag{17}$$

which is the relation between the orthogonal voltage instantaneous space vector and the current instantaneous space vector. In the same way we can show that

$$\widetilde{V} = 3(R + i\omega_L L)\widetilde{I}$$
 (18)

where R may be the blocked rotor equivalent resistance, the transmission line resistance or the sum of them (whatever is been considered). The instantaneous impedance are found to be

$$\dot{Z}_{inst} = 3 \left( R + j \, \omega_I \, L \, \right) \tag{19}$$

We can get the inductance L from (13)

$$L = \frac{Q}{3 \omega_I \left| \tilde{I} \right|^2} \tag{20}$$

and the resistance R from (10)

$$P_{R} = \frac{3}{2}R\left|\widetilde{I}\right|^{2} = Re\left\{\widetilde{S}\right\} - \frac{3}{2}L\left|\widetilde{I}\right| \frac{d\left|\widetilde{I}\right|}{dt}$$
 (21)

which is the resistive losses.

One may be interested on finding the Buchholz-Goodhue effective current calculated as the RMS of  $\left|\widetilde{I}\right|$  [4], during the entire motor acceleration time interval.

$$I_e = \sqrt{\frac{1}{T} \int_0^T |I|^2 dt} = \sqrt{\frac{\left\langle \left| \tilde{I} \right|^2 \right\rangle}{2}}$$
 (22)

In this way we may name  $\left|\widetilde{I}\right|$  as Buchholz-Goodhue

instantaneous effective current. The same is valid for voltages. Then the effective apparent power and effective power factor also applies for that time interval.

This concept for Buchholz-Goodhue effective quantities may be useful for varying load motors or intermittent motor operations and may be applied to any balanced load. The adequate time interval shall be in accordance to the time constant of the heat dissipation of each equipment of interest.

# VIII. APPENDIX B

Motor's data: 2250 HP, 60 Hz,

2400V (amplit.),  $1747 \, rpm$ ,  $J = 63.87 \, kgm^2$ ,

 $T_{load} \, = 9172 \, N.m \, , R_S \, = 0.0959 \, \Omega \, , R_R \, = 0.1339 \, \Omega \, , \label{eq:load}$ 

 $L_S = 0.5 \, mH$  ,  $L_R = 0.5 \, mH$  ,  $L_M = 26.45 \, mH$ 

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### X. BIOGRAPHIE

Dalgerti Lelis Milanez graduated from Escola de Engenharia de Lins in1972 as electrical engineer, received his Master an P.h.D. degrees from UNICAMP in 1986 and 1993, respectively. His employment experience includes the Sade-Sul Americana de Engenharia S.A and the Industrias Eternit SA, and presently he is at the Electrical Engineering Dep. at UNESP- São Paulo State University, in Ilha Solteira, SP, Brazil teaching electrical engineering and doing scientific research on power quality.